THE NONLINEAR THEORY OF ELECTROMAGNETIC WAVE ATTENUATION IN PLASMAS

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The absorption of a circularly polarized electromagnetic wave which propagates in a plasma along a magnetic field is analyzed. The exact equations of particle motion in the resonance region are solved with aid of elliptic functions. It is shown that the nonlinear damping constant has an oscillatory form. For $t \to 0$, it coincides with the constant obtained on the basis of linear theory, while for $t \to \infty$, in the absence of collisions, it tends to zero. The influence of collisions on wave absorption is studied. It is shown that with allowance for collisions, the damping constant depends on the amplitude of both the H_1 and $H_1^{-3/2}$ waves. The analysis of slowly decaying waves may be based on a model proposed by Dawson [1] and later modified in [2,3]. According to this model, all plasma particles are grouped into resonant and nonresonant ones. The velocity distribution function of the nonresonant particles is assumed to be the same as in the case of undamped waves. The distribution function of resonant particles at the initial instant is assumed to be Maxwellian. The nonlinear equations of motion of the resonant particles are integrated exactly. The damping constant is defined as the ratio of the energy expended by the wave at the resonant particles to the total energy of the wave.

In nonlinear formulation, resonant absorption appears to be nonstationary. After a time lapse on the order of several vibrational period of a particle captured by the wave, nonstationary absorption ceases, and stationary absorption, created by infrequent collisions, becomes essential. It is noteworthy that absorption of this type has been studied by V. E. Zakharov and V. I. Karpman [4] for the case of plasma waves.

1. For simplicity, thermal motion will be taken into account only for resonant particles. Nonresonant particles are considered to be cold. The dispersion equation for a circularly polarized electromagnetic wave that propagates in a cold plasma along the magnetic field has the form [5]

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_{0e}^2}{\omega(\omega \mp \omega_{He})} - \frac{\omega_{0i}^2}{\omega(\omega \mp \omega_{Hi})}. \qquad (1.1)$$

Let us examine a wave polarized in the direction of electron rotation. The lower sign in (1.1) refers to this wave. The analysis is limited to frequencies close to ω_{He}, i.e.,

$$\omega_{Hi} \ll \omega \lesssim |\omega_{He}|, \qquad (1.2)$$

thus, we assume that ions do not participate in the vibrations. Furthermore, there must be substantially fewer resonant than nonresonant particles,

$$|\omega_{He}| - \omega \gg kv_{Te}. \qquad (1.3)$$

Let us transfer to a system of coordinates that moves together with the wave. An electric field is absent in this system. The equations of motion of the particles (electrons) have the following form:

$$dv_x / dt = \omega_{H0}v_y + \omega_{H1}v_z \sin kz \quad (\omega_{H_0} = \omega_{He}),$$
$$dv_y / dt = -\omega_{H0}v_x + \omega_{H1}v_z \cos kz \ (\omega_{H1} = eH_1/mc),$$
$$dv_z / dt = -\omega_{H1}(v_x \sin kz + v_y \cos kz). \qquad (1.4)$$

Here, H_1 is the amplitude of the wave. The constant field H_0 is directed along the z-axis. By performing the change of variables

$$v_\xi = -\frac{k}{\omega_{H0}} v_x \cos kz - \frac{k}{\omega_{H0}} v_y \sin kz - \frac{\omega_{H1}}{\omega_{H0}},$$
$$v_\eta = -\frac{k}{\omega_{H0}} v_y \cos kz + \frac{k}{\omega_{H0}} v_x \sin kz,$$
$$v_\zeta = 1 - \frac{k}{\omega_{H0}} v_z, \qquad \tau = \omega_{H0}t, \qquad (1.5)$$

we obtain a system which is independent of z:

$$\frac{dv_\xi}{d\tau} = v_\eta v_\xi, \qquad \frac{dv_\zeta}{d\tau} = -\alpha - v_\xi v_\zeta,$$
$$\frac{dv_\zeta}{d\tau} = -\alpha v_\eta \quad \left(\alpha = \frac{\omega_{H1}}{\omega_{H0}} = \frac{H_1}{H^0}\right). \qquad (1.6)$$

The system of equations (1.6) has two simple integrals

$$p = v_\zeta^2 + 2\alpha v_\xi, \qquad q = v_\eta^2 + v_\xi^2 - 2v_\zeta. \qquad (1.7)$$

By eliminating v_η with the aid of (1.7) from the last equation of system (1.6), we arrive at a quadrature solution

$$\int_{v_\zeta}^{v_{\zeta 0}} \frac{dv_\zeta}{\sqrt{4q\alpha^2 - p^2 + 8\alpha^2 v_\zeta + 2pv_\zeta^2 - v_\zeta^4}} = \frac{\tau}{2}. \qquad (1.8)$$

The form of the response depends on the number of real roots of the equation

$$\chi \equiv v_\zeta^4 - 2pv_\zeta^2 - 8\alpha^2 v_\zeta + p^2 - 4q\alpha^2 = 0. \qquad (1.9)$$

Under the conditions of the problem studied, there can occur two cases: a) Eq. (1.9) has two real roots

$$v_{\zeta_1\zeta_2} = u \pm R^+(p, u); \qquad (1.10)$$

b) Eq. (1.9) has four real roots

$$v_{\zeta_1,\zeta_2} = u \pm R^+(p, u), \quad v_{\zeta_3\zeta_4} = -u \pm R^-(p, u),$$
$$R^\pm = (p - u^2 \pm 2\alpha^2 / u)^{1/2},$$
$$u^2 = {}^1/_3 p + {}^1/_2 [-r + {}^4/_3 \alpha^2 \sqrt{{}^1/_3 M}]^{1/3} +$$
$$+ {}^1/_2 [-r - {}^4/_3 \alpha^2 \sqrt{{}^1/_3 M}]^{1/3},$$
$$r = -{}^8/_{27} p^3 + {}^4/_3 \alpha^2 pq - 4\alpha^4,$$
$$s = {}^4/_3 \alpha^2 q - {}^4/_9 p^2, \ M = {}^{27}/_{16} (r^2 + s^2)\alpha^{-4}. \qquad (1.11)$$

To cases a and b in the pq-plane there correspond the regions

$$M(p, q) \equiv 4p^3 + 4\alpha^2 q^3 - 18\alpha^2 p^2 q^2 + 27\alpha^4 \gtrless 0. \qquad (1.12)$$

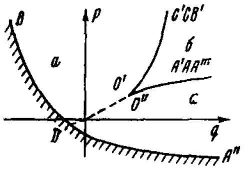

Fig. 1

In Fig. 1, curve A'O'C' which separates the regions which differ in the number of real roots of Eq. (1.9) is one of the two branches of the curve M(p, q) = 0; the second branch—the curve BDA"—limits the region of possible values of p and q. Substituting expressions (1.7) into M(p, q) = 0, we get an equation for the bounding surface in the space (v_ξ, v_η, v_ζ)

$$4(v_\zeta^2+2\alpha v_\xi^2)+4\alpha^2(v_\xi^2+v_\xi^2-2v_\zeta)^3-$$
$$-18\alpha^2(v_\zeta^2+2\alpha v_\xi)(v_\eta^2+v_\xi^2-2v_\zeta)-$$
$$-(v_\zeta^2+2\alpha v_\xi)_\eta(v_\eta^2+v_\xi^2-2v_\zeta)^2+27\alpha^4=0. \quad (1.13)$$

In the plane $v_\eta = 0$, Eq. (1.13) can be written in the form of the product

$$(\alpha+v_\xi v_\zeta)^2(4v_\zeta^3-4\alpha v_\xi^3+$$
$$+18\alpha v_\xi v_\zeta - v_\xi^2 v_\zeta^2+27\alpha^2)=0. \quad (1.14)$$

The surface (1.13) intersects the plane $v_\eta = 0$ along the curve

$$4v_\zeta^3-4\alpha v_\xi^3+18\alpha v_\xi v_\zeta - v_\xi^2 v_\zeta^2+27\alpha^2=0, \quad (1.15)$$

and is tangential to it along the hyperbola

$$\alpha+v_\xi v_\zeta=0. \quad (1.16)$$

Equation (1.18) is the equation of a nonlinear dipole with a potential energy $\chi(v_\zeta)$. The particles that correspond to hyperbola (1.16) are situated at the bottom of the potential well (Fig. 2); to curve (1.15), or more precisely to its segments O'A'" and O"A' (Fig. 3), there correspond particles with an infinite period (Fig. 4). It should be noted that particles which during their motion in velocity space intersect the plane $v_\eta = 0$ in the region A'"O'O"A' become trapped in the $v_\xi v_\eta$-plane, within a certain angle. They constitute an analog to particles trapped by a plasma wave.

Let us now analyze the solution of Eq. (1.8). In case a, it takes the form

$$v_\zeta=\frac{mv_{\zeta 1}-nv_{\zeta 2}}{n-m}+$$
$$+\frac{2nm(v_{\zeta 1}-v_{\zeta 2})}{n^2-m^2-(n-m)^2\operatorname{cn}[F(\varphi_0,k)-\tau\sqrt{nm},k]},$$
$$k^2=\frac{1}{2}+\frac{p-3u^2}{4\sqrt{(u^2+\alpha^2/u)^2-u^2(R^-)^2}},$$
$$n^2,m^2=u^2+\frac{\alpha^2}{u}\mp uR^+,$$
$$\varphi_0=2\operatorname{arc\,tg}\left[\frac{n}{m}\left(\frac{v_{\zeta 1}-v_{\zeta 0}}{v_{\zeta 0}-v_{\zeta 2}}\right)\right]^{1/2}. \quad (1.17)$$

Here, $F(\varphi, k)$ is an incomplete elliptic integral of the first kind, and $R^\pm$ is as defined in (1.11).

In case b, the solution depends also on the interval of integration over v_ζ:

$$v_\zeta=\frac{v_1(v_2-v_4)+v_4(v_1-v_2)\operatorname{sn}^2[F(\mu_0,r)+\tau\delta,r]}{v_2-v_4+(v_1-v_2)\operatorname{sn}^2[F(\mu_0,r)+\tau\delta,r]}$$

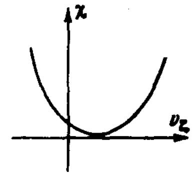

Fig. 2

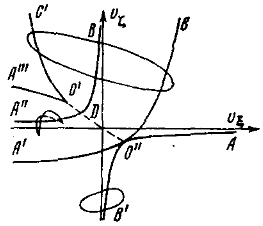

Fig. 3

$$(v_1<v_\zeta<v_2),$$
$$v_\zeta=\frac{v_3(v_2-v_4)-v_2(v_3-v_4)\operatorname{sn}^2[F(\lambda_0,r)+\tau\delta,r]}{v_2-v_4-(v_3-v_4)\operatorname{sn}^2[F(\lambda_0,r)+\tau\delta,r]}$$
$$(v_4<v_\zeta<v_3),$$
$$\mu_0=\arcsin\left(\frac{(v_2-v_4)(v_1-v_{\zeta 0})}{(v_1-v_2)(v_{\zeta 0}-v_4)}\right)^{1/2},$$
$$\lambda_0=\arcsin\left(\frac{(v_2-v_4)(v_3-v_{\zeta 0})}{(v_3-v_4)(v_2-v_{\zeta 0})}\right)^{1/2},$$
$$r=\{[(v_1-v_2)/(v_1-v_3)][(v_3-v_4)/(v_2-v_4)]\}^{1/2},$$
$$\delta={}^1/_4[(v_1-v_3)/(v_2-v_4)]^{1/2}. \quad (1.18)$$

Here

$$v_i=v_{\zeta i}\quad(i=1,2,3,4),\quad v_1>v_2>v_3>v_4.$$

For $\alpha/(p \ll 1,\ q \sim (kv_{Te}/\omega H_0)^2)^{1/2}$, the expression for v_ζ becomes appreciably simpler and has the same form both for case a and for case b:

$$v_\zeta\approx\sqrt{p}-\frac{3\alpha^2}{4\sqrt{p}}\left(\frac{v_{\xi 0}^2+v_{\eta 0}^2}{p}+\frac{4}{3\sqrt{p}}\right)+$$
$$+\left(\frac{\alpha^2 v_{\eta 0}^2}{p\sqrt{p}}+\frac{\alpha^2}{p}-\frac{\alpha v_{\zeta 0}}{\sqrt{p}}\right)\cos\tau\sqrt{p}-$$
$$-\left(\frac{\alpha v_{\eta 0}}{\sqrt{p}}+\frac{\alpha^2 v_{\xi 0}v_{\eta 0}}{4p\sqrt{p}}\right)\sin\tau\sqrt{p}+$$
$$+\frac{\alpha^2(v_{\xi 0}^2-v_{\eta 0}^2)}{4p\sqrt{p}}\cos 2\tau\sqrt{p}+$$
$$+\frac{\alpha^2 v_{\xi 0}v_{\eta 0}}{2p\sqrt{p}}\sin 2\tau\sqrt{p}, \quad (1.19)$$

where $v_{\xi 0}$, $v_{\eta 0}$, and $v_{\zeta 0}$ are the projections of the particle velocity onto the axes at the initial instant.

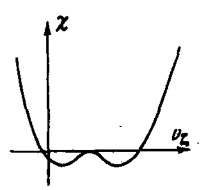

Fig. 4

2. The damping constant will be determined as the ratio of the energy expended by the wave at the resonant particles to the total energy E of the wave,

$$\gamma = \frac{1}{E}\iiint \frac{m}{2}\left[v_x^2 + v_y^2 + \left(v_z + \frac{\omega}{k}\right)^2\right]\frac{\partial f}{\partial t}\,dv_x dv_y dv_z\,,$$

$$E = \frac{H_1^2}{8\pi}\left(\frac{\omega}{kc}\right)^2\left[1 + \frac{k^2c^2}{\omega^2} + \frac{\omega_{0e}^2}{(\omega+\omega_{H0})^2}\right]. \qquad (2.1)$$

Assuming that the number of particles is conserved and that $v_x^2 + v_y^2 + v_z^2 = \text{const}$, we rewrite (2.1) as follows:

$$\gamma = \frac{1}{E}\iiint m v_z\left(\frac{\omega}{k}\right)\frac{\partial f}{\partial t}\,dv_x dv_y dv_z\,. \qquad (2.2)$$

The derivative $\partial f/\partial t$ will be determined from the kinetic equation, under the assumption that at the initial instant the particle velocity distribution function is Maxwellian. The expression for the damping constant takes the form

$$\gamma = -\frac{2m\omega_{H1}}{Ev_{Te}^2}\left(\frac{\omega}{k}\right)^2\left(\frac{\omega_{H0}}{k}\right)^2 \times$$
$$\times \iiint v_{\eta 0}\left(1 + \frac{kv_z}{\omega_{H0}}\right) f_0\,dv_x dv_y dv_z\,. \qquad (2.3)$$

Here, the value of f_0 in the integrand is taken only in the $v_z = (\omega + \omega_{H0})/k$ ($v_\zeta = 0$) plane. For cases a and b, we have

$$v_{\eta 0} = h\frac{\mathrm{sn}(F(\varphi, k) + \tau\sqrt{nm})\,\mathrm{dn}(F(\varphi, k) + \tau\sqrt{nm})}{\{[(n+m)/(n-m)] - \mathrm{cn}(F(\varphi, k) + \tau\sqrt{nm})\}^2}$$
$$(M(p, q) < 0),$$

$$v_{\eta 0} = l\frac{\mathrm{sn}(2F(\mu, r) - 2\tau\delta)\,\mathrm{dn}(2F(\mu, r) - 2\tau\delta)}{\{1 + [(v_1 - v_2)/(v_2 - v_4)]\,\mathrm{sn}^2(F(\mu, r) - 2\tau\delta)\}^2}$$
$$(M(p, q) > 0,\ v_2 < v_\zeta < v_1),$$

$$v_{\eta 0} = b\frac{\mathrm{sn}(2F(\lambda, r) - 2\tau\delta)\,\mathrm{dn}(2F(\lambda, r) - 2\tau\delta)}{1 - [(v_3 - v_4)/(v_2 - v_4)]\,\mathrm{sn}^2(F(\lambda, r) - 2\tau\delta)}$$
$$(M(p, q) > 0,\ v_4 < v_\zeta < v_3), \qquad (2.4)$$

$$h = \frac{2(nm)^{1/2}(v_2 - v_1)}{\alpha(n-m)^2},$$

$$l = \frac{(v_1 - v_4)(v_1 - v_2)}{4\alpha}\left(\frac{v_1 - v_3}{v_2 - v_4}\right)^{1/2}, \quad b = l\frac{(v_3 - v_4)}{(v_4 - v_1)}. \qquad (2.5)$$

For $\alpha/(\sqrt{p} \ll 1$, $q \sim (kv_{Te}/\omega_{H0})^2)$ the latter expression can be simplified as follows:

$$v_{\eta 0} \approx -\left(\frac{\alpha v_\eta^2}{p} + \frac{\alpha}{\sqrt{p}} - v_\xi\right)\sin\tau\sqrt{p} +$$
$$+ \left(v_\eta + \frac{\alpha v_\xi v_\eta}{4p}\right)\cos\tau\sqrt{p} -$$
$$- \frac{\alpha(v_\xi^2 - v_\eta^2)}{2p}\sin 2\tau\sqrt{p} - \frac{\alpha v_\xi v_\eta}{p}\cos 2\tau\sqrt{p} + \ldots \qquad (2.6)$$

Here, only the first-order terms with respect to $(\alpha/\sqrt{p})$ are retained.

First, we show that for $\tau \to 0$, a nonlinear damping constant reduces to a linear one. For small τ, particles with the smallest vibration period will engage in effective energy exchange with the wave. For these particles, the condition $\alpha/\sqrt{p} \ll 1$ is fulfilled. Hence, instead of $v_{\eta 0}$, its approximate expression from formula (2.6) can be substituted into (2.3). After some calculation, we get

$$\gamma = \omega_{0e}^2\exp\left[-\left(\frac{\omega + \omega_{H0}}{kv_{Te}}\right)^2\right] \times$$
$$\times\left[1 + \frac{k^2c^2}{\omega^2} + \left(\frac{\omega_{0e}}{\omega + \omega_{H0}}\right)^2\right]^{-1} \times$$
$$\times \frac{2}{\Pi^{1/2}kv_{Te}}\int_{\sqrt{p_1}}^{\infty}\frac{\sin(\sqrt{p}\,|\omega_{H0}|\,t)}{\sqrt{p}}\,d\sqrt{p}\,. \qquad (2.7)$$

Here, p_1 is the minimum value of p for which formula (2.6) is still applicable. Letting τ tend to zero, we arrive at a result from linear theory [5].

For an arbitrary instant the damping constant is expressed by triple integrals, which do not lend themselves to any essential simplification. Therefore, in order to determine the principal characteristics of this constant, we use the results of [2, 3] in which integrals of this type have been evaluated. If, for example, the new variables φ, k, q are substituted for v_x, v_y, v_z in (2.3), then φ will correspond to ξ, and k will approach α in formula (3.9) of [2].

In [2], it was shown that the damping constant will fluctuate with a characteristic time that differs by a factor on the order of unity from the vibrational period of particles situated at the bottom of the potential well. In the case under consideration, the velocity components of such particles satisfy the hyperbolic equation (1.16), in which case $v_\zeta > 0$, $v_\xi < -\alpha^{1/3}$.

By setting $v_{\xi 0} \approx kv_{Te}/\omega_{H0}$ in (1.17), we obtain the vibrational period of particles at the bottom of the potential well

$$T_0 = \frac{4\pi}{|\omega_{H0}|(nm)^{1/2}} \approx \frac{2\pi}{(kv_{Te}\omega_{H1})^{1/2}} \qquad (2.8)$$

Thus, as in the case of plasma waves, the characteristic time of the damping-constant fluctuations is inversely proportional to the square root of the wave amplitude. Hence, for small-amplitudes waves, processes in the resonant region proceed very slowly, the particle distribution function experiences almost no distorsion during a time on the order of the period of plasma oscillations ($T = 2\pi/\omega$), and a Maxwellian distribution function may be selected for the initial instant.

For large τ ($t \gg T_0$) the damping constant tends to zero, since the numerator of the integrand in (2.3) contains an elliptic function (2.4) which is integrated over its modulus. The decrease of the damping constant can be understood from simple physical considerations. The total energy of all particles situated on a trajectory with fixed p and q is conserved in the course of time if the distribution function depends only on these motion integrals. In the general case, the total energy varies between the maximum and minimum values which define p and q. At the initial instant, when the distribution function is Maxwellian, the total energy of most trajectories increases, and the wave attenuates. For large τ, owing to the difference in the periods, even such trajectories that are very similar with respect to p and q experience a phase shift. This means that the mean energy of all trajectories remains constant, and the wave ceases to attenuate. It should be noted that this result holds only in the absence of collisions, i.e., $\nu = 0$ (ν is the collision frequency). If $\nu \neq 0$, expression (2.3) is valid for times shorter than the particle collision time,

$$T_0 \ll t < 1/\nu\,. \qquad (2.9)$$

3. Owing to the collisions, the particle velocity distribution function will be partially restored, or, in other words, will become Maxwellian. For large $t \gg 1/\nu$, the plasma attains a steady state. The distribution function is determined in this case from a steady-

state kinetic equation containing a collision term, written in Landau's form

$$-v_\xi \frac{\partial f}{\partial \Psi} + \alpha v_\eta \frac{\partial f}{\partial v_\zeta} =$$

$$= \frac{\nu a}{2\pi |\omega_{H0}|} \frac{\partial}{\partial v_\xi}\left[\left(\frac{kv_{Te}}{\omega_{H0}}\right)^2 \frac{\partial f}{\partial v_\xi} + \left(v_\zeta - 1 + \frac{\omega}{|\omega_{H0}|}\right) f\right],$$

$$\Psi = \operatorname{arc\,tg} \frac{v_\eta}{v_\xi}, \qquad \nu = \frac{8\pi e^4 L k^3 n_0}{m^2 (|\omega_{H0}| - \omega)^3},$$

$$a = 1 + \left(\frac{\omega_{H0}}{kv_{Te}}\right)^2 [v_\eta^2 + (\alpha + v_\zeta)^2], \tag{3.1}$$

where ν has the meaning of the effective collision frequency in the resonant region, n_0 is the density, and L is the Coulomb logarithm.

It is convenient to change to the new variables θ and β

$$\theta = \pi - \psi, \qquad \beta = p + 2\alpha q,$$

$$q = v_\xi^2 + v_\eta^2 - 2 v_\zeta. \tag{3.2}$$

Since the distribution function is most sensitive to the longitudinal velocity and since $q \approx (kv_{Te}/\omega_{H0})^2$ in the resonant region, then Eq. (3.1) in the new variables takes the form

$$\frac{\partial f}{\partial \theta} = \frac{\nu a}{|\omega_{H0}|} \frac{\partial}{\partial \beta}\left\{\left(\frac{\omega}{|\omega_{H0}|} - 1\right) f + \right.$$

$$\left. + \sigma\left(\beta - 4\alpha \sqrt{q} \sin^2 \frac{\theta}{2}\right)^{1/2} \left[2\left(\frac{kv_{Te}}{\omega_{H0}}\right)^2 \frac{\partial f}{\partial \beta} + f\right]\right.,$$

$$\sigma = \operatorname{sgn} v_\zeta, \quad a = 1 + q(\omega_{H0}/kv_{Te})^2. \tag{3.3}$$

This equation differs from Eq. (1.8) in [4] only in the symbols employed. With the aid of the last equation in system (1.4), we calculate the energy expended by the wave per unit time at particles with a fixed q,

$$\frac{dW}{dt} = -\frac{1}{8} m\omega \left|\frac{\omega_{H0}}{k}\right|^5 \times$$

$$\times \int_{-\pi}^{\pi} d\theta \int_{\Delta(\theta)}^{\infty} \sum_\sigma f_\sigma \Delta(\theta) [\beta - \Delta(\theta)]^{-1/2},$$

$$\Delta(\theta) = 4\alpha \sqrt{q} \sin^2 (1/2) \theta. \tag{3.4}$$

With the aid of the results obtained in [4], we rewrite expression (3.4) in the form

$$\frac{dW}{dt} = \frac{1}{4}\left(\frac{\pi}{2}\right)^{1/2} \frac{m n_0 \omega \nu |\omega_{H0}| a}{k^4 v_{Te}^2} \alpha^{1/4} q^{1/8} c_0 \times$$

$$\times \exp\left[-\frac{c_0^2}{2} - \frac{q}{2}\left(\frac{\omega_{H0}}{kv_{Te}}\right)^2\right],$$

$$c_0 = \frac{|\omega_{H0}| - \omega}{kv_{Te}}. \tag{3.5}$$

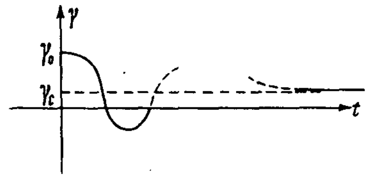

Fig. 5

By integrating (3.5) over q and dividing by the total wave energy, we obtain the damping constant

$$\gamma_c = \frac{7}{8} \sqrt{\pi}\, \Gamma\left(\frac{1}{4}\right) \frac{\omega \nu n_0 m c_0}{k^4} \sqrt{\frac{kv_{Te}}{|\omega_{H1}|}} \exp\left(-\frac{c_0^2}{2}\right) \times$$

$$\times \left\{\frac{H_1^2}{8\pi}\left(\frac{\omega}{kc}\right)^2 \left[1 + \left(\frac{kc}{\omega}\right)^2 + \left(\frac{\omega_{0e}}{\omega + \omega_{H0}}\right)^2\right]\right\}^{-1}. \tag{3.6}$$

In the case of stationary absorption, γ_c depends on the amplitude of both the H_1 and $H_1^{-3/2}$ wave. Generally speaking, formula (3.6) holds for $t \gg 1/\nu$. A general expression describing the damping constant with allowance for collisions at any instant could not be derived. A qualitative plot of γ vs. t is shown in Fig. 5.

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